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Rolling Solution for Tachyon Condensation in Open String Field Theory

Liudmila Joukovskaya*
DAMTP, Centre for Mathematical Sciences, University of Cambridge,
Wilberforce Road, Cambridge CB3 0WA, UK

Abstract

Open string field theory in the level truncation approximation is considered. It is shown that the energy conservation law determines the existence of rolling tachyon solution. The coupling of the open string field theory action to a Friedmann-Robertson-Walker metric is considered and as a result the new time dependent rolling tachyon solution is presented and possible cosmological consequences are discussed.

1 Introduction

Consideration of fundamental theories such as M/String Theory in the cosmological context continues to attract attention in the literature.

One of the interesting questions is the role of the tachyon in String Theory and Cosmology. The great progress in our understanding about tachyon condensation was made in the past decade[2, 1] but a lot of interesting issues are still open. Among the most important ones is a better understanding of the dynamics in tachyon condensation process.

Open String Field Theory (OSFT) [3] gives us a tachyon effective action[4, 5] which is derived from first principles and correctly describes tachyon physics and could represent most exhaustive framework for studying tachyon dynamics.

The tachyon dynamics even in the case without gravity represents a nontrivial task which was widely studied [6, 8, 9, 7, 10], while considering OSFT coupled to the gravity in a Friedmann-Robertson-Walker (FRW) type of metric makes this problem more nontrivial because of the d'Alambert operators in curved spaces that would appear in the action. This fact makes mathematical investigation of this problem much more complicated but will benefit us with desired time-dependent solution interpolating between different vacua.

*E-mail: `l.joukovskaya@damtp.cam.ac.uk`

In this work new numerical rolling solution for tachyon condensation will be presented. We will show that consideration of OSFT coupled to the gravity which represents more realistic situation from cosmological point of view allows the existence of the rolling tachyon configuration which was forbidden by energy conservation law in the case without gravity.

2 The Model

The action of bosonic cubic string field theory has the form

$$S = -\frac{1}{g_0^2}\int (\frac{1}{2}\Phi\cdot Q_B\Phi + \frac{1}{3}\Phi\cdot(\Phi * \Phi)), \tag{1}$$

where g_0 is the open string coupling constant, Q_B is BRST operator, $*$ is noncommutative product and Φ is the open string field containing component fields which correspond to all the states in string Fock space.

Considering only tachyon field $\phi(x)$ at the level (0,0) the action (2) becomes

$$S = \frac{1}{g_0^2}\int d^{26}x\left[\frac{\alpha'}{2}\phi(x)\Box\phi(x) + \frac{1}{2}\phi^2(x) - \frac{1}{3}K^3\Phi^3(x) - \Lambda\right], \tag{2}$$

where α' is the Regge slope, $K = \frac{3\sqrt{3}}{4}$, ϕ is a scalar field, $\Phi = e^{k\Box_g}\phi$, $k = \alpha'\ln K$, $\Box = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$ and $\Lambda = \frac{1}{6}K^{-6}$ was added to the potential to set the local minimum of the potential to zero according Sen's conjecture [11].
The action (2) leads to equation of motion

$$(\alpha'\Box + 1)e^{-2k\Box}\Phi = K^3\Phi^2. \tag{3}$$

The Stress Tensor for our system is[1]

$$T_{\alpha\beta}(x) = -g_{\alpha\beta}\left(\frac{1}{2}\phi^2 - \frac{\alpha'}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{3}K^3\Phi^3 - \Lambda\right) - \alpha'\partial_\alpha\phi\partial_\beta\phi \tag{4}$$

$$-g_{\alpha\beta}\,k\int_0^1 d\rho\left[(e^{k\rho\Box}K^3\Phi^2)(\Box e^{-k\rho\Box}\Phi) + (\partial_\mu e^{k\rho\Box}K^3\Phi^2)(\partial^\mu e^{-k\rho\Box}\Phi)\right]$$

$$+2k\int_0^1 d\rho\ (\partial_\alpha e^{k\rho\Box}K^3\Phi^2)(\partial_\beta e^{-k\rho\Box}\Phi).$$

The energy is defined as $E(t) = T^{00}$ and pressure as $P(t)_i = -T^i_i$ (no summation) and for our system are

$$\mathcal{E} = \mathcal{E}_k + \mathcal{E}_p + \Lambda + \mathcal{E}_{nl1} + \mathcal{E}_{nl2}, \quad \mathcal{P} = \mathcal{E}_k - \mathcal{E}_p - \Lambda - \mathcal{E}_{nl1} + \mathcal{E}_{nl2}$$

[1]Note that here and below integration over ρ is understood as limit of the following regularization

$$\int_0^1 d\rho f(\rho) = \lim_{\epsilon_1\to+0}\lim_{\epsilon_2\to+0}\int_{\epsilon_1}^{1-\epsilon_2} d\rho f(\rho).$$

where

$$\mathcal{E}_k = \frac{\alpha'}{2}(\partial\phi)^2, \quad \mathcal{E}_p = -\frac{1}{2}\phi^2 + \frac{K^3}{3}\Phi^3$$

$$\mathcal{E}_{nl1} = k\int_0^1 d\rho\left(e^{k\rho\Box}K^3\Phi^2\right)\left(-\Box e^{-k\rho\Box}\Phi\right),$$

$$\mathcal{E}_{nl2} = -k\int_0^1 d\rho\left(\partial e^{k\rho\Box}K^3\Phi^2\right)\left(\partial e^{-k\rho\Box}\Phi\right).$$

In this paper we will be interested in spatially homogeneous configurations for which Beltrami-Laplace operator used above takes the form $\Box_g = -\partial^2$. To avoid calculation of $e^{-k\rho\partial^2}$ term which is much harder to compute then $e^{k\rho\partial^2}$ $(k > 0)$ as computation of the former results in an ill-posed problem we will use the following representation for nonlocal energy terms $\mathcal{E}_{nl1}$ and $\mathcal{E}_{nl2}$ which are valid on the equation of motion for the scalar field

$$\mathcal{E}_{nl1} = k\int_0^1 d\rho\left((-\alpha'\partial^2+1)e^{(2-\rho)k\partial^2}\Phi\right)\left(\partial^2 e^{k\rho\partial^2}\Phi\right),$$

$$\mathcal{E}_{nl2} = -k\int_0^1 d\rho\left(\partial(-\alpha'\partial^2+1)e^{(2-\rho)k\partial^2}\Phi\right)\left(\partial e^{k\rho\partial^2}\Phi\right).$$

Claim 1[2] The Energy

$$E = \frac{\alpha'}{2}(\partial\phi)^2 - \frac{1}{2}\phi^2 + \frac{K^3}{3}\Phi^3 + \Lambda + k\int_0^1 d\rho\ ((-\alpha'\partial^2+1)e^{(2-\rho)k\partial^2}\Phi)\overleftrightarrow{\partial}(\partial e^{k\rho\partial^2}\Phi),$$

is conserved on the solutions of equation of motion (3)

$$(-\alpha'\partial^2+1)e^{2k\partial^2}\Phi = K^3\Phi^2$$

where $A\overleftrightarrow{\partial}B = A\partial B - B\partial A$.

Proof.

$$\frac{dE(t)}{dt} = \alpha'\partial\phi\partial^2\phi - \phi\partial\phi + \Phi^3\partial\Phi + k\int_0^1 d\rho((-\alpha'\partial^2+1)e^{(2-\rho)k\partial^2}\Phi)\overleftrightarrow{\partial^2}(\partial e^{k\rho\partial^2}\Phi).$$

Using following identity[18]

$$\int_0^1 d\rho(e^{\rho\partial^2}\varphi)\overleftrightarrow{\partial^2}(e^{(1-\rho)\partial^2}\phi) = \varphi\overleftrightarrow{e^{\partial^2}}\phi,$$

equation of motion and definition of field Φ, we have

$$\frac{dE(t)}{dt} = \alpha'\partial\phi\partial^2\phi - \phi\partial\phi + \Phi^3\partial\Phi + \partial\Phi\overleftrightarrow{e^{k\partial^2}}(\alpha'\partial^2-1)e^{k\partial^2}\Phi = \partial\Phi\left[\Phi^3 + (\alpha'\partial^2-1)e^{2k\partial^2}\Phi\right] = 0.$$

[2]Similar theorem was proved in [6], but the variant we use is more useful for numerical calculations, because in order to define action of the exponential operator we need to do only one well defined integration instead of the summation over infinite series expansions with which one always need to be very careful about the convergence and related issues.

It is straightforward to generalize this statement to arbitrary potential and only finite number of fields. Let us consider physical consequence of the energy conservation law.

Claim 2[3] There doesn't exist continuous solution of equation (3) which satisfies the boundary conditions

$$\lim \Phi(t) = \begin{cases} 0, & t \to \infty, \\ K^{-3}, & t \to -\infty \end{cases} \tag{5}$$

or vice-versa (in terms $t \to -t$).

Proof. Let us assume existence of such solution and calculate energy at the extremum points, we get $E(\Phi = 0) = \Lambda$ and $E(\Phi = K^{-3}) = -\frac{1}{6}K^{-6} + \Lambda$, i.e. energy values at $t \to +\infty$ and $t \to -\infty$ are different what contradicts the energy conservation theorem.

As we can see energy conservation law plays crucial role in the existence of the time dependent solutions of equation of motion for the case of level truncation approximation for OSFT. The above statement could probably be generalized to the case of full OSFT action because for the action with cubic interaction solution interpolating between maximum and minimum in the effective potential has to interpolate between vacua with different energy.

3 The Model Coupled to the Gravity

In this section we consider more realistic case when gravity is included

$$S = \frac{1}{g_0^2} \int d^4x \sqrt{-g} \left(\frac{m_p^2}{2} R + \frac{1}{2}\phi \Box_g \phi + \frac{1}{2}\phi^2 - \frac{1}{3}K^3\Phi^3 - \Lambda \right), \tag{6}$$

here $m_p^2 = g_0^2 M_{pl}^2$ and we will work in units where $\alpha' = 1$. As a particular metric we will consider a FRW one

$$ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2),$$

for which the Beltrami-Laplace operator for spatially-homogeneous configurations takes the form $\Box_g = -\partial^2 - 3H(t)\partial = -\mathcal{D}_{\mathrm{H}}^2$. Scalar field and Friedmann equations are

$$(-\mathcal{D}_{\mathrm{H}}^2 + 1)e^{2k\mathcal{D}_{\mathrm{H}}^2}\Phi = K^3\Phi^2, \tag{7}$$

$$3H^2 = \frac{1}{m_p^2}\,\mathcal{E}, \quad 3H^2 + 2\dot{H} = -\frac{1}{m_p^2}\,\mathcal{P}. \tag{8}$$

Inclusion the gravity drastically changes the question of existence of the dynamical interpolation between maximum and minimum of the scalar field potential. This happens because then there are no restrictions from energy conservation law.

[3] The similar claim for the p-adic string model was proved in [6], which rules out the possibility that the tachyon may roll monotonically down from one extremum reaching the tachyon vacuum.

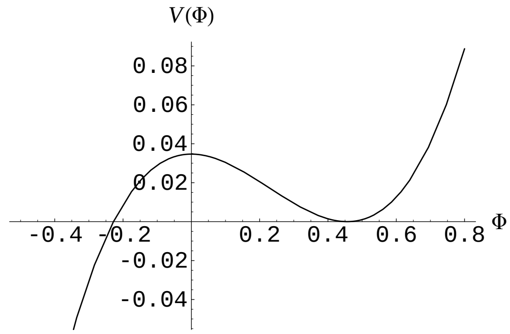

Figure 1: The potential

According to the Friedmann and scalar field equations we can expect the scalar field rolling solution and Hubble function satisfying the following boundary conditions

$$\lim \Phi(t) = \begin{cases} 0, & t \to \infty, \\ K^{-3}, & t \to -\infty \end{cases} \qquad \lim H(t) = \begin{cases} (18K^6)^{-1/2}, & t \to \infty, \\ 0, & t \to -\infty \end{cases} \tag{9}$$

or vice-versa (in terms of $t \to -t$). Note that from cosmological perspective we are interested only in positive values for the Hubble function and thus we do not consider negative sign in front of the square root in (9).

To analyze physical situation let us consider potential in which motion is expected. Naive extraction of potential from the model action (6) results in $V(\Phi) = -\frac{1}{2}\Phi^2 + \frac{1}{3}K^3\Phi^3 + \Lambda$. The constant Λ represents the D-brane tension which according to Sen's conjecture must be added to cancel the negative energy appearing due to the presence of tachyon. We have obtained two type of solutions. The first one is an ordinary rolling solution which starts from $\Phi = 0$ and goes towards configuration $\Phi = K^{-3}$ which is associated with the true vacuum. This solution can be interpreted as a description of the D-brane decay. The second one is a rolling solution which goes in the opposite direction, which appears in this model possibly because of the non-locality in the interaction. It is known that nonlocal dynamics has many interesting properties which are not present in the local case. In particular the "slop effect" [6, 8, 14] which is present in the obtained solutions (Fig. 2, 3) when the scalar field goes beyond the values from which the scalar field configuration starts – situation which is not possible in the local models. Potentially a similar effect can initiate non-symmetry in the potential in ekpyrotic [15] and cyclic cosmology [16].

4 Numerical Scheme for Solution Construction

For numerical calculations we operate with scalar field equation of motion (7) and the difference of equations (8)

$$(-\mathcal{D}_{\mathrm{H}}^2 + 1)e^{2k\mathcal{D}_{\mathrm{H}}^2}\Phi = K^3\Phi^2, \quad \dot{H} = -\frac{1}{2m_p^2}\,(\mathcal{P} + \mathcal{E}). \tag{10}$$

The outline of the numerical scheme is the following[4]

- For equations (10) we introduce lattice in t variable and then solve resulting system of nonlinear equations using iterative relaxation solver using discrete L_2 norm to control error tolerance.

- The nontrivial thing from computational point of view is efficient evaluation of $e^{2k\rho\mathcal{D}^2_{\mathrm{H}}}\Phi$ for $\rho \in [0,2]$. This operator could be interpreted in terms of initial value problem for the following diffusion equation with boundary conditions

$$\partial_\rho\varphi(t,\rho) = \partial_t^2\varphi(t,\rho) + 3H(t)\partial_t^2\varphi(t,\rho), \tag{11}$$

$$\varphi(0,t) = \Phi(t), \quad \varphi(\rho,\pm\infty) = \Phi(\pm\infty).$$

  Once solution of this equation is constructed we have $e^{2k\rho\mathcal{D}^2_{\mathrm{H}}}\Phi(t) = \varphi(\rho,t)$.

- To solve (11) we used second order Crank-Nicholson scheme which is based on approximation

$$e^{2k\Delta_\rho\tilde{\mathcal{D}}^2_{\mathrm{H}}}\varphi = \left(1 + k\Delta_\rho\tilde{\mathcal{D}}^2_{\mathrm{H}}\right)\left(1 - k\Delta_\rho\tilde{\mathcal{D}}^2_{\mathrm{H}}\right)^{-1}\varphi + o(\Delta_\rho^2\|\tilde{\mathcal{D}}^2_{\mathrm{H}}\|),$$

  where $\tilde{\mathcal{D}}^2_{\mathrm{H}}$ is a $\mathcal{D}^2_{\mathrm{H}}$ operator on the t-lattice (it thus has a finite norm) and Δ_ρ is a step size along ρ variable. Derivatives in t variable were approximated using 4th order finite differences on uniform lattice (symmetric scheme).

- In order to exclude possible artifacts of this specific numerical scheme we tried Chebyshev-pseudospectral method which is known to have impressive exponential convergence [17]. This scheme is known to have very different properties [17] compared to finite difference scheme described above, but it produced the same results up to the approximation error which gives us confidence in the existence of the rolling solution reported in this work.

5 Rolling Tachyon Solution and their Cosmological Consequences

Solutions of (7) and (8) are presented on Fig. 2 and 3. As we can see we obtained accelerating rolling solutions for tachyon scalar field Φ. It is natural to address cosmological issues in the context of rolling tachyon solution [19]. Taking into account that acceleration

[4] It is important that the described below scheme is general and was also used for solving cosmological equations for the case of Cubic Fermionic Field Theory with the quartic interaction term in (6) instead of cubic one as well as for p-adic string model at least for $p = 2,3$. Obtained interpolating solutions between corresponding maximum and minimum of the tachyon potential look very similar to one which will be presented in section 5 and will be presented in [18]. It looks that cosmology effaces difference between cubic and quartic interaction for the type of solutions indicated above.

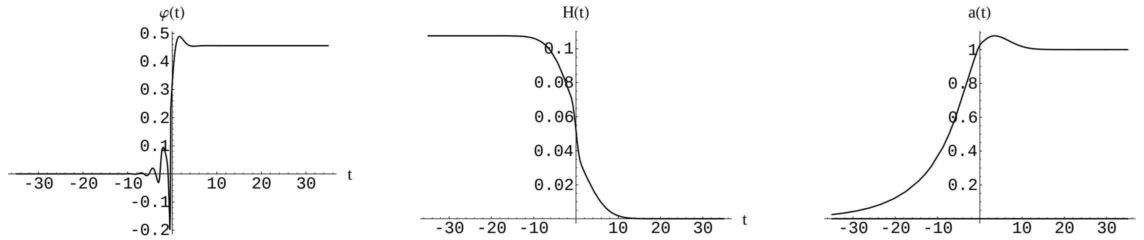

Figure 2: Solutions of the scalar field (7) and Friedmann equation (8) Φ, H and a (left to right) for $m_p^2 = 1$.

of the Universe is one of the most fascinating processes of the modern cosmology many authors tried to explain among other possibilities with a possible explanation via scalar field. It is interesting that cosmology gives us this solution owing to coupling our action (2) to a FRW metric and the consequent inclusion of a Hubble friction term which leads to time-dependent rolling solution with exponentially decreasing oscillations around the minimum. Moreover because generally speaking string scale does not exactly coincide with Plank mass we obtain some freedom in settling m_p^2 parameter for numerical calculations which enters into Friedmann equations and as a result govern the value of Hubble function $H(t)$. Thus decreasing the value of m_p^2 leads to more smooth profile for rolling solution while increasing m_p^2 results in higher oscillations of the solution in comparison to those presented on the Fig. 2 and 3, more details will be presented in [18]. During the process of completion of this work appeared [10] in which OSFT tachyon in the dilaton background was considered and time-like rolling tachyon solution were obtained. Because dilaton appears from the same string sector as graviton including the dilaton into the tachyon action can qualitatively reproduce behavior of the tachyon in the curved spaces.

Concluding this section we would like to summarize that we obtained time dependent accelerating solution interpolating between unstable and the true vacua (see Fig. 2) which can be interpreted as being responsible for acceleration of the Universe during this rolling from unstable vacuum to the true vacuum, after which it disappears. Evolution of the scalar field in the opposite direction is also possible with the Hubble function in form of increasing kink when scale factor $a(t)$ starts from the constant plateau and exponentially grows, which seems counter intuitive but can be related to late time acceleration.

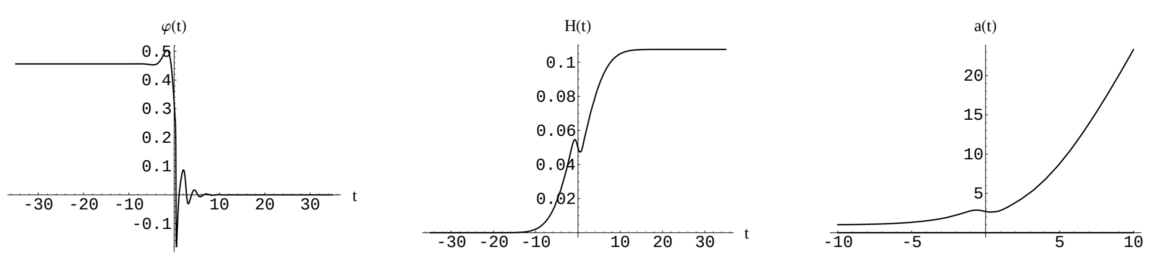

Figure 3: Solutions of the scalar field (7) and Friedmann equation (8) Φ, H and a (left to right) for $m_p^2 = 1$.

6 Conclusion

The Witten's cubic open bosonic string filed theory in the level truncation approximation was considered. It was shown that the energy conservation law determines existence of rolling tachyon solution. As a result it was explicitly shown that the non-existence of the rolling solution in the Minkowski case is a necessary consequence of the energy conservation law of the system. The modification of conservation law in the presence of the gravity is discussed. The first rolling solution for tachyon condensation in this theory is presented and possible cosmological consequences are discussed. Although only lowest excitation in the full OSFT were taken into account there are solid reasons to suppose that the general picture for the tachyon condensation process will be the same in the case of full OSFT.

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